– Discrete Mathematics –

May 2023

Name: ..... Total: .....

Exercise 1. *A survey of 60 college students was taken asking about which meals they regularly ate at school. Thirty students usually ate breakfast at school, 26 usually ate lunch there and 23 usually ate dinner at school. Six students said they ate all three meals at school. Eleven ate both breakfast and lunch there whereas 15 only ate breakfast at school and 12 only ate lunch there.*

(a) *How many students ate both lunch and dinner at school?*
(b) *How many didn't eat any meals at school?*
(c) *How many ate exactly one meal at school?*

(4 points)

Exercise 2. *How many* 3*-element subsets does the set*

$$A = \left\{u^2 - v^2 : u \in \{-1, 0, 1, 2\}, v \in \{0, 1\}\right\}$$

have? *(4 points)*

Exercise 3. *Use the Euclidean algorithm to find* $\gcd(2369, 3933)$ *and compute integers* x *and* y *for which*

$$2369x + 3933y = \gcd(2369, 3933).$$

(4 points)

Exercise 4. *Expand the following expression using the binomial theorem:*

$$\left(-\frac{2x}{3} + \frac{3}{2x}\right)^3.$$

(4 points)

Exercise 5. *Determine all non-negative integral solutions of the equation*

$$13x + 44y = 654.$$

(4 points)

Exercise 6. *Describe the value(s) of* k *for which* $k\binom{15}{k}$ *is largest.* *(4 points)*

Exercise 7. *a. Determine the decimal representation of the following numbers.*

$2030_4 =$ 10

$2030_7 =$ 10

$2030_8 =$ ..... 10 *(6 points)*

b. Determine the appropriate representations of the following numbers.

$577_{10} =$ 5

$1719_{10} =$ 7

$1507_{10} =$ ..... 9 *(6 points)*

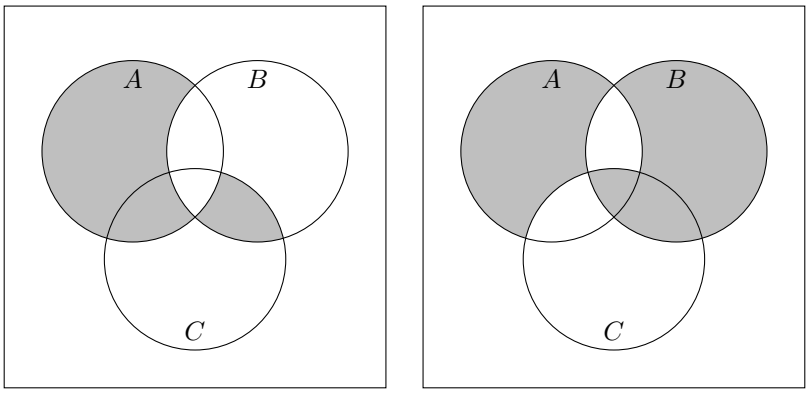
Exercise 8. *Suppose* a_n *is a sequence such that* $a_1 = 3, a_2 = 21$ *and* $a_{n+2} = 2a_{n+1} + 3a_n$ *for all* $n \geq 1$. *Prove that* $a_n = 2 \times 3^n + 3 \times (-1)^n$. *(8 points)*

Discrete Mathematics

December 11, 2017

Name: ..... Total: .....

Exercise 1. *Use set notation to describe the shaded areas:* *(4 point)*



Exercise 2. *How many* 3*-element subsets does the set* $A = \{(u-v)^2 : u, v \in \{-1, 1, 2\}\}$ *have?* *(4 point)*

Exercise 3. *Use the Euclidean algorithm to find* $\gcd(1079, 689)$. *(4 point)*

Exercise 4. *Expand the following expression using the binomial theorem:*

$$\left(\frac{x}{z} - \frac{z}{y}\right)^3.$$

(4 point)

Exercise 5. *How many solutions does the equation* $x_1 + x_2 + x_3 + x_4 + x_5 = 4$ *have, where* x_1, x_2, x_3, x_4, x_5 *are integers such that* $x_i \geq 3 - i, i = 1, 2, \ldots, 5$? *(4 point)*

Exercise 6. *(a) How many nine digit numbers can be formed from the digits 1,2,3,1,1,2,2,3,1?*
(b) How many six digit numbers can be formed from the digits 0,1,1,1,1,2? *(4 points)*

Exercise 7. *a. Determine the decimal representation of the following numbers.*

$1112_4 =$ ..... $_{10}$

$1112_5 =$ ..... $_{10}$

$1112_6 =$ ..... $_{10}$ *(6 points)*

b. Determine the appropriate representations of the following numbers.

$1211_{10} =$ ..... $_5$

$1211_{10} =$ ..... $_7$

$1211_{10} =$ ..... $_9$ *(6 points)*

Exercise 8. *Prove by induction that* 7 *divides* $13^n - 6^n$ *for all* $n \in \mathbb{N}$. *(8 points)*