Discrete Mathematics

9th December, 2019

Name: ..... Total: .....

Exercise 1. *How many subsets does the set* $A = \{-1, 2, \{-1, 2\}, \{3\}, \{-1, 2, 3\}\}$ *have?*

Exercise 2. *How many* 3*-element subsets does the set* $A = \{u + v : u \in \{1, 2, 3\}, v \in \{1, 2\}\}$ *have?* *(4 point)*

Exercise 3. *Use the Euclidean algorithm to find* $\gcd(2929, 2407)$. *(4 point)*

Exercise 4. *Expand the following expression using the binomial theorem:*

$$\left(\frac{x}{2} - 1\right)^4.$$

(4 point)

Exercise 5. *How many solutions does the equation* $x_1+x_2+x_3+x_4+x_5+x_6 = 23$ *have, where* $x_1, x_2, x_3, x_4, x_5, x_6$ *are integers such that* $x_k \geq (-1)^k + k$? *(4 point)*

Exercise 6. *Describe all values of* n *and* k *for which*

$$\binom{n}{k+1} = 9\binom{n}{k}.$$

(4 points)

Exercise 7. *a. Determine the decimal representation of the following numbers.*

$103_7 =$ ..... $_{10}$

$103_8 =$ ..... $_{10}$

$103_9 =$ ..... $_{10}$ *(6 points)*

b. Determine the appropriate representations of the following numbers.

$2019_{10} =$ ..... $_3$

$2019_{10} =$ ..... $_6$

$2019_{10} =$ ..... $_9$ *(6 points)*

Exercise 8. *Prove that*

$$\sum_{k=1}^{n}(9 - 2k) = -n^2 + 8n$$

for every positive integer n. *(8 points)*

Discrete Mathematics

4 December 2023

Name: ..... Total: .....

1. Draw a Venn diagram for the following sets:
   $A \cup B \cup C = \{2, 8, 3, 24, 25, 50, 48\}$
   A contains even numbers,
   B contains numbers divisible by 3,
   C contains numbers divisible by 5.

   (4 points)

2. How many subsets does the set $A = \{0, 1, \{1\}, 3, \{1, 3\}\}$ have?

   (4 points)

3. Use the Euclidean algorithm to find $\gcd(a, b)$ and compute integers x and y for which
   $$ax + by = \gcd(a, b) :$$
   where $a = 2071, b = 1178$.

   (4 points)

4. How many solutions does the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 2$ have, where x_1, x_2, x_3, x_4, x_5 are integers such that $x_i \geq -i + 3$?

   (4 points)

5. How many seven digit numbers can be formed from the digits 0,2,2,2,4,2,3?

   (4 points)

6. Describe all values of n and k for which
   $$\binom{n}{k+1} = 13\binom{n}{k}.$$

   (4 points)

7. a. Determine the decimal representation of the following numbers.

   (6 points)

   $1332_4 =$ ..... $_{10}$

   $1332_5 =$ ..... $_{10}$

   $1332_6 =$ ..... $_{10}$

   b. Determine the appropriate representations of the following numbers.

   (6 points)

   $1047_{10} =$ ..... $_6$

   $1047_{10} =$ ..... $_7$

   $1047_{10} =$ ..... $_8$

8. Prove by induction that 15 divides $4^{2n} - 1$ for any positive integer n.

   (8 points)