Stellingen

Propositions belonging to the thesis
Effective Methods for Diophantine Equations
by Sz. Tengely

We denote by $\mathbb{Z}, \mathbb{N}, \mathbb{Q}$ and $\overline{\mathbb{Q}}$ the ring of rational integers, the set of natural numbers, the field of rationals and the field of algebraic numbers, respectively.

1. Let $q > 1$ be an integer and $f : \mathbb{N} \longrightarrow \overline{\mathbb{Q}}$ a periodic function mod q, i.e. $f(n+q) = f(n)$ for all $n \in \mathbb{N}$. Denote by $\varphi(q)$ the Euler totient function and by $\nu_p(n)$ the exponent to which p divides n. Put

$$P(d) = \{p \text{ prime } \mid p \text{ divides } q, \nu_p(d) \geq \nu_p(q)\},$$
$$\varepsilon(r,p) = \nu_p(q) + \frac{1}{p-1} \text{ if } p \in P(r) \text{ and } \nu_p(r) \text{ otherwise.}$$

Let $f(m) = f(n)$ for all m, n with $\nu_p(m) = \nu_p(n)$ for all prime divisors p of q. Then $\sum_{n=1}^{\infty} \frac{f(n)}{n} = 0$ if and only if

$$\sum_{v|q} \varphi\left(\frac{q}{v}\right) f(v) = 0$$

and

$$\sum_{r=1}^{q} f(r)\varepsilon(r,p) = 0 \quad \text{for all prime divisors } p \text{ of } q.$$

Literature: T. Okada, *On a certain infinite series for a periodic arithmetical function*, Acta Arith., **40** (1981/82), 143-153.

2. Erdős conjectured that if $f : \mathbb{N} \longrightarrow \mathbb{Z}$ is periodic mod q such that $f(n) \in \{-1, 1\}$ when $n = 1, \ldots, q-1$ and $f(q) = 0$, then $\sum_{n=1}^{\infty} \frac{f(n)}{n} \neq 0$. However, there exists a function $f : \mathbb{N} \longrightarrow \{\pm 1\}$ with period 36 such that

$$\sum_{n=1}^{\infty} \frac{f(n)}{n} = 0.$$

Literature: A. E. Livingston, *The series $\sum_1^{\infty} f(n)/n$ for periodic f*, Canad. Math. Bull., **8** (1965), 413-432.

3. If $f : \mathbb{N} \longrightarrow \mathbb{Z}$ is a function with period $q = p_1^{\alpha_1} \cdots p_r^{\alpha_r}$ such that $f(n) \in \{-1, 1\}$ when $n = 1, \ldots, q-1$ and $f(q) = 0$ and $f(m) = f(n)$ for all m, n with $\nu_p(m) = \nu_p(n)$ for all primes $p \mid q$ and $\sum_{n=1}^{\infty} \frac{f(n)}{n} = 0$. Then $\alpha_i \geq 2$ for $i = 1, 2, \ldots, r$.

4. Let $U = \{u_1, \ldots, u_k\}$ be a set of distinct positive integers and $s = \sum_{i=1}^{k} u_i$. The set U is said to be a unique-sum set if the equation $\sum_{i=1}^{k} c_i u_i = s$ with $c_i \in \mathbb{N} \cup \{0\}$ has only the solution $c_i = 1$ for $i = 1, 2, \ldots, n$. Let u be an element of a unique-sum set U. Then
$$\#U \leq \frac{u}{2} + 1.$$

5. For every positive integer n the set
$$G_n = \bigcup_{k=0}^{n-1} \{2^n - 2^k\}$$
is a unique-sum set.

6. All the solutions of the Diophantine equation
$$x^4 + 2x^3 - 9x^2y^2 + 2xy - 15y - 7 = 0$$
in rational integers are given by
$$(x, y) \in \{(-4, -1), (-1, -1), (1, -1), (2, -1)\}.$$

7. There exists a solution of the Diophantine equation $x^2 + q^4 = 2y^p$ in positive integers x, y, p, q, with p and q odd primes.

8. The Diophantine equation $x^2 + q^{2m} = 2 \cdot 2005^p$ does not admit a solution in integers x, m, p, q, with p and q odd primes.

9. Let C be the curve given by
$$Y^2 = X^6 - 17X^4 - 20X^2 + 36.$$
Then $C(\mathbb{Q}) = \{\infty^-, \infty^+, (\pm 1, 0), (0, \pm 6)\}$.

10. One can use TEX not only for typesetting but also for resolving Diophantine equations.

11. Klaar is kész.