– Discrete Mathematics –

November 2021

Name: ..... Total: .....

Exercise 1. *Draw a Venn diagram for the following sets:*
$A \cup B \cup C = \{4, 6, 7, 9, 12, 14, 21, 35\}$
A contains even numbers,
B contains numbers divisible by 3,
C contains numbers divisible by 7. *(4 points)*

Exercise 2. *How many* 3*-element subsets does the set*

$$A = \{u + v : u \in \{-1, 0, 1, 2\}, v \in \{0, 1\}\}$$

have? *(4 points)*

Exercise 3. *Use the Euclidean algorithm to find* $\gcd(2291, 1363)$. *(4 points)*

Exercise 4. *Expand the following expression using the binomial theorem:*

$$\left(-\frac{x}{3} + 3\right)^3.$$

(4 points)

Exercise 5.
(a) How many eight digit numbers can be formed from the digits 1,1,1,2,4,3,3,4?

(b) How many seven digit numbers can be formed from the digits 2,3,1,2,3,0,4? *(4 points)*

Exercise 6. *Describe all values of* n *and* k *for which*

$$\binom{n}{k+1} = 4\binom{n}{k}.$$

(4 points)

Exercise 7. *a. Determine the decimal representation of the following numbers.*

$210_3 =$ ..... $_{10}$

$210_4 =$ ..... $_{10}$

$210_5 =$ ..... $_{10}$ *(6 points)*

b. Determine the appropriate representations of the following numbers.

$2121_{10} =$ ..... $_7$

$2121_{10} =$ ..... $_8$

$2121_{10} =$ ..... $_9$ *(6 points)*

Exercise 8. *Prove by induction that*

$$\sum_{k=1}^{n} \frac{k+2}{k(k+1)2^k} = 1 - \frac{1}{(n+1)2^n}$$

for any positive integer n. *(8 points)*

– Discrete Mathematics –

April, 2022

Name: ..... Total: .....

Exercise 1. *Suppose 135 students are surveyed about their favorite ice cream flavors. Draw a Venn diagram for the following statements, and then use it to answer the questions. 63 like Chocolate, 61 like Caramel, 53 like Vanilla, 28 like Caramel and Vanilla, 31 like Chocolate and Caramel, 22 like Chocolate and Vanilla, 15 like all three flavors.*

How many students do not like any of the given flavors?

How many students like only Caramel? *(4 points)*

Exercise 2. *How many* 4*-element subsets does the set*

$$A = \left\{u^2 + uv - v^2 : u \in \{-1, 0, 1, 2\}, v \in \{0, 1\}\right\}$$

have? *(4 points)*

Exercise 3. *Use the Euclidean algorithm to find* $\gcd(2369, 3933)$ *and compute integers* x *and* y *for which*

$$2369x + 3933y = \gcd(2369, 3933).$$

(4 points)

Exercise 4. *Expand the following expression using the binomial theorem:*

$$\left(-\frac{2x}{3} + \frac{3}{x}\right)^3.$$

(4 points)

Exercise 5. *Determine all non-negative integral solutions of the equation*

$$13x + 44y = 654.$$

(4 points)

Exercise 6. *Determine the value(s) of* k *for which* $k\binom{44}{k}$ *is largest.* *(4 points)*

Exercise 7. *a. Determine the decimal representation of the following numbers.*

$2030_4 =$ 10

$2030_7 =$ 10

$2030_8 =$ ..... 10 *(6 points)*

b. Determine the appropriate representations of the following numbers.

$577_{10} =$ 5

$1719_{10} =$ 7

$1507_{10} =$ ..... 9 *(6 points)*

Exercise 8. *Suppose* a_n *is a sequence such that* $a_1 = 3, a_2 = 21$ *and* $a_{n+2} = 2a_{n+1} + 3a_n$ *for all* $n \geq 1$*. Prove that* $a_n = 2 \times 3^n + 3 \times (-1)^n$*.* *(8 points)*