Discrete Mathematics

December 2, 2014

1. Provide three sets A, B and C which satisfy the following cardinality conditions
$$|A \cap B \cap C| = 1,$$
$$|A \cap B| = 2, \quad |A \cap C| = 2, \quad |B \cap C| = 3,$$
$$|A| = 4, \quad |B| = 5, \quad |C| = 5.$$
(4 points)

2. Expand the following expressions using the binomial theorem.
(a) $(-2x + x^2)^2$,
(b) $(x - y + 2)^3$.
(4 points)

3. How many 5-element subsets does the set $A = \{-3, -2, -1, 0, 1, 2, 3\}$ have?
(4 points)

4. Apply the Euclidean algorithm to determine $\gcd(2923, 1739)$.
(4 points)

5. How many 8-digit numbers can be made out of the digits $1, 2, 2, 3, 3, 3, 4, 4$?
(4 points)

6. What is the number of solutions of the equation
$$x_1 + x_2 + x_3 + x_4 = 12,$$
where x_1, x_2, x_3, x_4 are integers such that $x_i \geq 2i - 3$ for $i = 1, 2, 3, 4$?
(4 points)

7. a. Determine the decimal representation of the following numbers.
(6 points)

$654_7 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots_{10}$

$654_8 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots_{10}$

$654_9 = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots_{10}$

b. Determine the appropriate representations of the following numbers.
(6 points)

$654_{10} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots_{7}$

$654_{10} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots_{8}$

$654_{10} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots_{9}$

8. Let the sequence T_n defined by $T_1 = T_2 = T_3 = 1$ and $T_n = T_{n-1} + T_{n-2} + T_{n-3}$ for $n \geq 4$. Prove that
$$T_n < 2^n, \quad n \in \mathbb{N}.$$
(8 points)

Discrete Mathematics

10th December, 2019

Name: .... Total: ....

Exercise 1. *Provide three sets A, B and C which satisfy the following cardinality conditions*

$$|A \cap B \cap C| = 0,$$
$$|A \cap B| = 2, \quad |A \cap C| = 2, \quad |B \cap C| = 2,$$
$$|A| = 5, \quad |B| = 6, \quad |C| = 7.$$

(4 points)

Exercise 2. *How many 2-element subsets does the set $A = \{u + v : u \in \{1, 3, 5\}, v \in \{2, 4\}\}$ have? (4 points)*

Exercise 3. *Use the Euclidean algorithm to find* $\gcd(2263, 1457)$. *(4 points)*

Exercise 4. *Expand the following expression using the binomial theorem:*

$$\left(-\frac{x}{2} + 2\right)^4.$$

(4 points)

Exercise 5. *How many solutions does the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 25$ have, where x_1, x_2, x_3, x_4, x_5 are integers such that $x_k \geq (-1)^k + k$?* *(4 points)*

Exercise 6. *Describe all values of n and k for which*

$$\binom{n}{k+1} = 10\binom{n}{k}.$$

(4 points)

Exercise 7. *a. Determine the decimal representation of the following numbers.*

$120_5 =$ .... $_{10}$

$120_6 =$ .... $_{10}$

$120_7 =$ .... $_{10}$ *(6 points)*

b. Determine the appropriate representations of the following numbers.

$2019_{10} =$ .... $_4$

$2019_{10} =$ .... $_5$

$2019_{10} =$ .... $_7$ *(6 points)*

Exercise 8. *Prove by induction that 6 divides*

$$\sum_{k=0}^{2n-1} 5^k$$

for any positive integer n. *(8 points)*