Discrete Mathematics

6 November 2024

Exercise 1. *Let* $A = \{1, 2, 3, 6, 7, 8, 9\}$ *and* $B = \{2, 3, 4, 6, 8\}$. *What are the elements of the set*

$$C = (A \setminus B) \cup (B \setminus A)?$$

Exercise 2. *Let* $A = \{1, 2, 3, 4, 6, 7, 8\}, B = \{2, 4, 5, 6, 8\}$ *and* $C = \{1, 2, 3, 4, 7\}$. *What are the elements of the set* $(A \cap B) \setminus (C \cap B)$*?*

Exercise 3. *Draw a Venn diagram for the following sets:*
(a) $(A \cap B) \cup (C \setminus B)$,
(b) $(A \setminus B) \cup (A \setminus C)$,
(c) $(A \cup B) \cap (B \cup C)$,
(d) $(A \cap B) \cup (B \cap C) \cup (C \setminus A)$.

Exercise 4. *Provide three sets* A, B *and* C *which satisfy the following cardinality conditions*

$$|A \cap B \cap C| = 1,$$
$$|A \cap B| = 2, \quad |A \cap C| = 2, \quad |B \cap C| = 2,$$
$$|A| = 3, \quad |B| = 4, \quad |C| = 5.$$

Exercise 5. *How many subsets does the set* $A = \{-1, 1, \{-1, 3\}, \{1, 2\}, \{-1, 1, 2, 3\}\}$ *have?*

Exercise 6. *How many* 4*-element subsets does the set* $A = \{-1, 1, \{1, 2\}, 2, \{6, 7\}, 8\}$ *have?*

Exercise 7. *Expand the following expressions using the binomial theorem.*
(1) $(2 - x + x^2)^2$,
(2) $(-2 + 3x)^3$,

Exercise 8. *Find all values of* n *and* k *for which*

$$\binom{n}{k+1} = 20\binom{n}{k}.$$

Exercise 9. *a. Determine the decimal representation of the following numbers.*

$345_7 =$ 10

$345_8 =$ 10

$345_9 =$ 10

b. Determine the appropriate representations of the following numbers.

$543_{10} =$ 7

$543_{10} =$ 8

$543_{10} =$ 9

Exercise 10. *How many solutions does the equation* $x_1+x_2+x_3+x_4 = 3$ *have, where* x_1, x_2, x_3, x_4 *are integers such that* $x_i \geq -i$*?*

Exercise 11. *How many seven digit numbers can be formed from the digits 1,1,1,5,3,3,3?*

Exercise 12. *How many eight digit numbers can be formed from the digits 0,1,1,1,4,3,3,3?*

Exercise 13. *Suppose* a_n *is a sequence such that* $a_{n+2} = a_{n+1} - a_n$ *for all* $n \geq 1$. *Given that* $a_{17} = 7$ *and* $a_{21} = 2$, *find* a_1.

Exercise 14. *Prove that*

$$5^n + 2 \times 11^n$$

is a multiple of 3 for every positive integer n.

Exercise 15. *Prove by induction that*

$$2^n < n!$$

for any positive integer $n \geq 4$.