– Discrete Mathematics –

May 2023

Name: .... Total: ....

Exercise 1. *A survey of 60 college students was taken asking about which meals they regularly ate at school. Thirty students usually ate breakfast at school, 26 usually ate lunch there and 23 usually ate dinner at school. Six students said they ate all three meals at school. Eleven ate both breakfast and lunch there whereas 15 only ate breakfast at school and 12 only ate lunch there.*

(a) *How many students ate both lunch and dinner at school?*
(b) *How many didn't eat any meals at school?*
(c) *How many ate exactly one meal at school?*

(4 points)

Exercise 2. *How many* 3*-element subsets does the set*

$$A = \left\{u^2 - v^2 : u \in \{-1, 0, 1, 2\}, v \in \{0, 1\}\right\}$$

have? *(4 points)*

Exercise 3. *Use the Euclidean algorithm to find* $\gcd(2369, 3933)$ *and compute integers* x *and* y *for which*

$$2369x + 3933y = \gcd(2369, 3933).$$

(4 points)

Exercise 4. *Expand the following expression using the binomial theorem:*

$$\left(-\frac{2x}{3} + \frac{3}{2x}\right)^3.$$

(4 points)

Exercise 5. *Determine all non-negative integral solutions of the equation*

$$13x + 44y = 654.$$

(4 points)

Exercise 6. *Describe the value(s) of* k *for which* $k\binom{15}{k}$ *is largest.* *(4 points)*

Exercise 7. *a. Determine the decimal representation of the following numbers.*

$2030_4 =$ .... $_{10}$

$2030_7 =$ .... $_{10}$

$2030_8 =$ .... $_{10}$ *(6 points)*

b. Determine the appropriate representations of the following numbers.

$577_{10} =$ .... $_5$

$1719_{10} =$ .... $_7$

$1507_{10} =$ .... $_9$ *(6 points)*

Exercise 8. *Suppose* a_n *is a sequence such that* $a_1 = 3, a_2 = 21$ *and* $a_{n+2} = 2a_{n+1} + 3a_n$ *for all* $n \geq 1$. *Prove that* $a_n = 2 \times 3^n + 3 \times (-1)^n$. *(8 points)*

Discrete Mathematics

April 12, 2016

1. Provide three sets A, B and C which satisfy the following cardinality conditions
$$|A \cap B \cap C| = 2,$$
$$|A \cap B| = 2, \quad |A \cap C| = 3, \quad |B \cap C| = 3,$$
$$|A| = 4, \quad |B| = 5, \quad |C| = 5.$$

(4 points)

2. Expand the following expressions using the binomial theorem.
   (a) $(xy - yz + zx)^2$,
   (b) $(2x - 3y)^3$.

(4 points)

3. How many subsets does the set $A = \{1, 2, \{1\}, \{\{1\}\}, \{2\}, \{1, 2\}\}$ have?

(4 points)

4. Apply the Euclidean algorithm to determine $\gcd(2701, 3737)$ and a solution $(x, y) \in \mathbb{Z}^2$ of the equation
$$2701x + 3737y = \gcd(2701, 3737).$$

(4 points)

5. How many 9-digit numbers can be made out of the digits $0, 1, 1, 3, 3, 3, 3, 9, 9$?

(4 points)

6. What is the number of solutions of the equation
$$x_1 + x_2 + x_3 + x_4 + x_5 = 16,$$
where x_1, x_2, x_3, x_4, x_5 are integers such that $x_i \geq (-1)^i + i$ for $i = 1, 2, 3, 4, 5$?

(4 points)

7. a. Determine the decimal representation of the following numbers.

(6 points)

$2332_5 =$ .... $_{10}$

$2332_6 =$ .... $_{10}$

$2332_7 =$ .... $_{10}$

b. Determine the appropriate representations of the following numbers.

(6 points)

$2332_{10} =$ .... $_7$

$2332_{10} =$ .... $_8$

$2332_{10} =$ .... $_9$

8. Prove that
$$\frac{10^n + 3 \cdot 4^{n+2} + 5}{9}$$
is an integer for all $n \in \mathbb{N}$.

(8 points)