Discrete Mathematics

April 25, 2017

Name: ..... Total: .....

1. Provide three sets A, B and C which satisfy the following cardinality conditions

$$|A \cap B \cap C| = 0,$$
$$|A \cap B| = 2, \quad |A \cap C| = 4, \quad |B \cap C| = 3,$$
$$|A| = |B| = 7, \quad |C| = 8.$$

(4 points)

2. How many 4-element subsets does the set $A = \{-1, 1, \{1, 2\}, 2, \{6, 7, 8\}, 8\}$ have?

(4 points)

3. Draw a Venn diagram for the following sets:
   (a) $(A \cup B) \cup (C \setminus B)$,
   (b) $(A \cup B) \cap (A \setminus C)$,
   (c) $(A \cup B) \setminus (B \cup C)$,
   (d) $(A \setminus B) \cup (B \setminus C) \cup (C \cap A)$.

(4 points)

4. Use the Euclidean algorithm to find $\gcd(a, b)$ and compute integers x and y for which

$$ax + by = \gcd(a, b) :$$

where $a = 2201, b = 1333$.

(4 points)

5. How many solutions does the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 5$ have, where $x_1, x_2, x_3, x_4, x_5, x_6$ are integers such that $x_i \geq -i + 3$?

(4 points)

6. Expand the following expression using the binomial theorem:

$$\left(3x - \frac{y}{x}\right)^3.$$

(4 points)

7. a. Determine the decimal representation of the following numbers.

(6 points)

$1144_5 =$ 10

$1144_6 =$ 10

$1144_7 =$ 10

b. Determine the appropriate representations of the following numbers.

(6 points)

$765_{10} =$ 9

$765_{10} =$ 7

$765_{10} =$ 6

8. Let $a_0 = 1, a_1 = 4$ and $a_n = 6a_{n-1} - 9a_{n-2}, n \geq 2$. Prove by induction that $a_n = 3^n + n3^{n-1}$ for all integers $n \geq 0$.

(8 points)

Discrete Mathematics

December 10, 2018

Name: ..... Total: .....

1. Provide three sets A, B and C which satisfy the following cardinality conditions

$$|A \cap B \cap C| = 2,$$
$$|A \cap B| = 2, \quad |A \cap C| = 4, \quad |B \cap C| = 2,$$
$$|A| = 5, \quad |B| = 3, \quad |C| = 5.$$

(4 points)

2. How many 3-element subsets does the set $A = \{-1, 1, 3, \{-1\}, \{1, 3\}, \{-1, 3\}\}$ have?

(4 points)

3. How many subsets does the set $A = \{-1, 2, \{-1, 2\}, \{3\}, \{-1, 2, 3\}\}$ have?

(4 points)

4. Use the Euclidean algorithm to find $\gcd(1133, 913)$ and compute integers x and y for which

$$1133x + 913y = \gcd(1133, 913).$$

(4 points)

5. How many solutions does the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 6$ have, where $x_1, x_2, x_3, x_4, x_5, x_6$ are integers such that $x_i \geq 3 - i, i \in \{1, 2, 3, 4, 5, 6\}$?

(4 points)

6. How many eight digit numbers can be formed from the digits 1,1,2,2,3,3,4,0?

(4 points)

7. a. Determine the decimal representations of the following numbers.

(6 points)

$315_6 =$ 10

$315_7 =$ 10

$315_8 =$ 10

b. Determine the appropriate representations of the following numbers.

(6 points)

$789_{10} =$ 5

$789_{10} =$ 7

$789_{10} =$ 9

8. Prove by induction that

$$\sum_{k=1}^{n} (6 - k) = \frac{n(11 - n)}{2}$$

for any positive integer n.

(8 points)