Discrete Mathematics
26/03/2019

Exercise 1. *Let* $A = \{1, 2, 3, 5, 6, 7, 9\}$ *and* $B = \{2, 3, 4, 5, 8\}$*. What are the elements of the set* $C = (A \setminus B) \cup (B \setminus A)$*?*

Exercise 2. *How many* 4*-element subsets does the set* $A = \{-1, 1, \{1\}, 2, \{6, 7\}, 8, 11\}$ *have?*

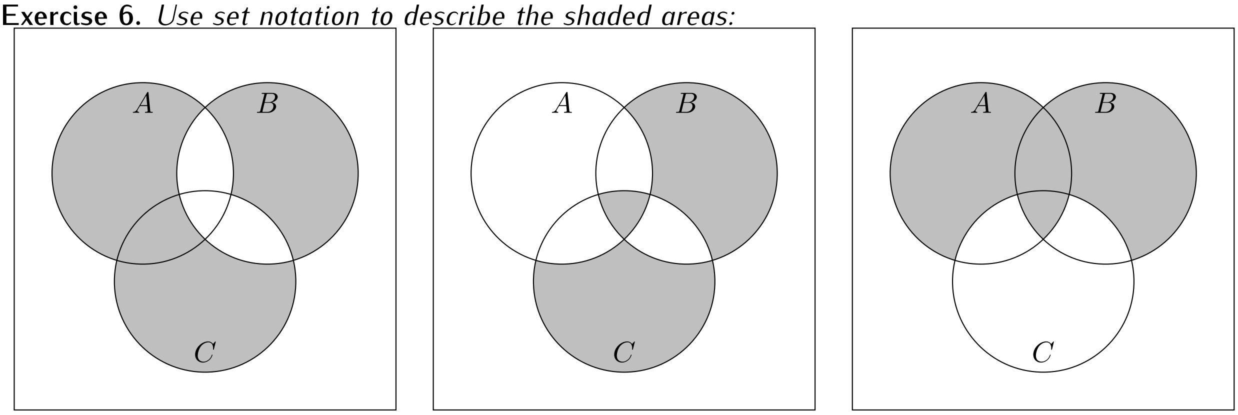
Exercise 3. *Draw a Venn diagram for the following sets:*
$A \cup B \cup C = \{4, 8, 9, 10, 12, 15, 18, 25, 32, 35, 70\}$
A *contains even numbers,*
B *contains numbers divisible by 3,*
C *contains numbers divisible by 5.*

Exercise 4. *Provide three sets* A, B *and* C *which satisfy the following cardinality conditions*

$$|A \cap B \cap C| = 2,$$
$$|A \cap B| = 2, \quad |A \cap C| = 3, \quad |B \cap C| = 4,$$
$$|A| = 4, \quad |B| = 6, \quad |C| = 7.$$

Exercise 5. *Draw a Venn diagram for the following sets:*
(a) $(A \setminus (B \cup C)) \cup (B \cap C)$,
(b) $(A \setminus B) \cap (A \cup (C \setminus B))$,
(c) $(A \cup B) \setminus (B \cap C)$,
(d) $((A \cap B) \cup (B \setminus C)) \setminus (C \cap A)$.

Exercise 6. *Use set notation to describe the shaded areas:*



Exercise 7. *Use the Euclidean algorithm to find* $\gcd(a, b)$ *and compute integers* x *and* y *for which*

$$ax + by = \gcd(a, b) :$$

(a) $a = 1122, b = 154$
(b) $a = 2233, b = 374$.

Exercise 8. *Expand the following expressions using the binomial theorem.*
(1) $(-x - 3y + 4z)^2$,
(2) $(x - \frac{3}{x})^4$.

Exercise 9. *Determine all non-negative integral solutions of the equation*

$$7x + 19y = 234.$$

Exercise 10. *(a) How many eight digit numbers can be formed from the digits 1,1,1,2,4,3,3,4?*
(b) How many seven digit numbers can be formed from the digits 0,2,1,1,2,3,4?

Exercise 11. *Determine the appropriate representation of the following numbers.*

$1122_4 =$ .... $_7$
$246_8 =$ .... $_7$

Exercise 12. *Prove that $n^3 + 2n$ is divisible by 3 for all positive integers n.*

Exercise 13. *Find the value of k for which $k\binom{71}{k}$ is largest.*

Exercise 14. *Describe all values of n and k for which*

$$\binom{n}{k+1} = 16\binom{n}{k}.$$

Exercise 15. *(a) How many solutions does the equation $x_1 + x_2 + x_3 + x_4 = 5$ have, where x_1, x_2, x_3, x_4 are integers such that $x_i \geq -i - 1$?*

(b) How many solutions does the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 5$ have, where x_1, x_2, x_3, x_4, x_5 are integers such that $x_i \geq -i + 1$?

Exercise 16. *Prove that*

$$11^n - 4^n$$

is a multiple of 7 for every positive integer n.

Exercise 17. *Prove by induction on n that 13 divides $2^{4n+2} + 3^{n+2}$ for all natural numbers n.*

Exercise 18. *Using induction, show that $4^n + 15n - 1$ is divisible by 9 for all $n \geq 1$.*

Exercise 19. *Prove by induction that*

$$\sum_{k=1}^{n} (-1)^k k^2 = \frac{(-1)^n n(n+1)}{2}$$

for $n \in \mathbb{N}$.

Exercise 20. *Prove that 21 divides $4^{n+1} + 5^{2n-1}$ for all positive integer n.*

Exercise 21. *Show that for each $n \geq 2$*

$$\sum_{k=1}^{n} \frac{1}{\sqrt{k}} > \sqrt{n}.$$