

Algorithms in Algebra and Number Theory

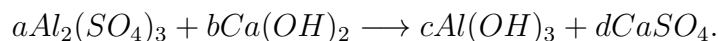
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Exercise 1. Let $M = \begin{pmatrix} 1 & -1 \\ 6 & -4 \end{pmatrix}$. Provide functions $f_1(n), f_2(n), f_3(n), f_4(n)$, such that

$$M^n = \begin{pmatrix} f_1(n) & f_2(n) \\ f_3(n) & f_4(n) \end{pmatrix}.$$

Determine three different prime values of n , for which $f_1(n)$ is a prime.

Exercise 2. Balance the following chemical equation:



Exercise 3. By means of the algebraic method determine a 3-coloring of the graph $G = \text{graphs.PetersenGraph}()$

such that the vertices 0, 7 and 8 are colored in the same way.

Exercise 4. Let $f(x) = x^6 + 5x^2 + x + 3$ be a polynomial over \mathbb{F}_p , for some prime p . Determine the largest two digit prime for which the polynomial is reducible by applying the Berlekamp's algorithm. What is the value of p ?

Exercise 5. Solve the discrete logarithm problem $23^x \equiv 57 \pmod{557}$ by means of the modified Pollard's- ρ method in which the sequence is defined by $x_0 = 1$ and

$$x_{n+1} = \begin{cases} gx_n & \text{if } x_n \equiv 0 \pmod{3}, \\ ghx_n & \text{if } x_n \equiv 1 \pmod{3}, \\ x_n^2 & \text{if } x_n \equiv 2 \pmod{3}. \end{cases}$$